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ORIGINAL ARTICLE

Nanoscale trajectory planning with flexible Acc/Dec and look-ahead method

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Abstract The importance of look-ahead and flexible acceleration and deceleration (Acc/Dec) technology to high-precision and high-speed computer numerical control (CNC) technology was firstly discussed; the massive data computing and high-precision requirement problem in NC blocks' trajectory planning was revealed. Motived by this, the flexible ACC/DEC and look-ahead trajectory planning method was systematically studied. A four-level trajectory planning strategy including position, velocity, acceleration, and jerk planning was addressed in detail. A corner speed control method suitable for any multi-axis was expounded, and a classification method in forward speed planning was researched to describe various speed profiles. On the above basis, a nanoscale trajectory planning method using the long blocks segmentation method was proposed. The overall effectiveness of the proposed strategy is demonstrated by the simulation and machining of a group of identical impellers.

Keywords Trajectory planning · Flexible Acc/Dec · Look-ahead · NANO interpolation

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1 Introduction

With the continuous development of computer numerical control (CNC) technology, the CNC system has not only been used in the control of conventional lathe, three-axis milling machine, but also more and more widely used in five-axis machine tools, compound machine tools, and flexible machine cells. The high-precision and high-speed machining technology, multi-axis linkage control technology, and nanometer interpolation technology has become an important direction of CNC technology [1–3]. In complex surface machining, such as die and mold machining, fast feed-rates were selected to travel along the tool path consisted of numerous short linear segments. The shorter the line, the smaller the approximation error will be, which has resulted in a large number of sequential small segments, many of which are only a few microns long. In this case, considering the requirements of Acc/Dec in the trajectory planning [4], if the CNC system executes each block just line by line without pre-reading the following blocks, the actual feedrate will be less than the programmed feedrate and the feedrate at the corners between two blocks become discontinuous. This leads to a very frequent start-stop process, not only reduces the machining efficiency, but also inevitably lead to lower surface quality [5–8]. To solve the above problem, modern CNC systems should have the look-ahead function which can pre-read hundreds of blocks and calculates an adequate feedrate for each axis within the maximum allowable feedrate and Acc/Dec; however, the function requires much computing power. Recently, with the advance of CPU power, the number of the blocks that can be used for look-ahead has grown to a thousand [9], which can only be used in high-end CNC systems.

In order to reduce the vibration of the machine tools during machining, the flexible Acc/Dec control technology such as the S-curve Acc/Dec which can realize the jerk control is also used in modern CNC systems [10]. But this is computationally

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expensive [11–14]. Meanwhile, the length of the sequential linear path in the trajectory planning has a large variation range, maybe from 10000 to 0.001 mm. In nanometer, the range is from -1×10^{11} to 1×10^{11} nm. In binary mode, it is -1×2^{37} to 1×2^{37}. If considering the intermediate storage in multiplication, the range should be from -1×2^{74} to 1×2^{74}; a 75-bit integer space should be allocated for each direction of the tool position. As we all know, today's industrial processors are mostly less than 64 bits; the double-precision floating-point number is only 64 bits. Although we can realize large integer operation with 32-bit or 64-bit processors through indirect algorithm, the efficiency of the computing process will be reduced. So in high-end CNC systems, special designed processors are usually used to realize the large bit integer computing, which brings additional hardware cost and difficulty of CNC system development.

With respect to realization of efficient flexible Acc/Dec and look-ahead technology in high-speed and high-precision machining, a generalized look-ahead feedrate planning algorithm is proposed in [12], which can correctly reconfigured with different Acc/Dec schemes, but the S-curve Acc/Dec was not addressed in detail, and the accuracy control problem also was not discussed. A whole S-curve speed control algorithm with look-ahead method was developed in [15], and further implemented on a five-axis machining center in [13]. The continuous S-curve Acc/Dec method was also introduced and realized in a pipe cutting CNC system in [14] and used in the NURBS interpolation in [16]. But all of them did not make a complete classification of the S-curve Acc/Dec in look-ahead mode and also did not give detailed steps to compute the speed profile parameters [17]. Trying to use the flexible Acc/Dec with look-ahead method in the five-axis CNC machining of dual NURBS makes the trajectory planning even more complex and fails to give full consideration to problems of computational efficiency. According to the above analysis, the authors mentioned mainly focus on the generation of whole S-curve speed profiles and did not give enough consideration to the computation of all possible S-curve Acc/Dec profiles and the algorithm's calculation efficiency. While in actual processing, for the real-time requirement of the machining process, the flexible Acc/Dec with look-ahead method contains many of the special Acc/Dec cases and needs numerous arithmetic operations.

This paper will focus on the efficient implementation technology of the trajectory planning with flexible Acc/Dec and look-ahead method. Firstly, basic methods of trajectory planning including S-curve Acc/Dec, blocks look-ahead, and four-level planning including position planning, speed planning, acceleration planning, and jerk planning method will be systematically studied. Secondly, the continuous S-curve Acc/Dec control method with look-ahead function will be detailed including the corner speed control method, backward and forward speed-profile generation method, in which a speed-profile classification method of continuous blocks will be discussed. Thirdly, a nanoscale trajectory planning method using long blocks segmentation method will be proposed based on the analysis of the characteristics of the whole trajectory planning's calculation process. Finally, simulation and experiment will be conducted to verify that the proposed strategy is capable of generate continuous S-curve speed-profile and efficiently accomplish complex surface machining tasks.

2 Basic methods of trajectory planning

2.1 S-curve Acc/Dec

As shown in Fig. 1, a complete S-curve Acc/Dec feedrate profile should include seven sub-phases [14, 18]: Acc-Acc, constant Acc, Dec-Acc, constant velocity, Acc-Dec, constant Dec, and Dec-Dec. The first three sub-phases are the acceleration process, and the last three sub-phases are the deceleration process. The symbols in Fig. 1 are defined as follows: t represents time, s represents displacement, v represents velocity, a represents acceleration, and J represents jerk; v_s is the starting velocity of one program block; $v_{\max}$ is the maximum allowable speed, and v_e is the ending velocity; $a_{\max}$ is the maximum allowable acceleration during acceleration process, and $d_{\max}$ is the maximum allowable deceleration during deceleration process; $J_{\max}$ is the maximum allowable jerk; $t_k(k=0,1,\cdots,7)$ is the time boundaries of each sub-phase, where $t_0=0$; $\tau_k(k=1,\cdots,7)$ denotes the relative time that starts at the beginning of the $k-th$ sub-phase, and $\tau_k=t-t_{k-1}, t_{k-1}\le\tau_k\le t_k$; $T_k(k=1,\cdots,7)$ is the duration of the k-th sub-phase, where $T_k=t_k-t_{k-1}$.

For a whole S-curve Acc/Dec, it should have $T_1=T_3, T_5=T_7$, i.e., the Acc-Acc time should equal Dec-Acc, and the Acc-Dec time should equal Dec-Dec, so as to maintain a smooth transition of the acceleration. Let $T_{\mathrm{aj}}=T_1=T_3, T_{\mathrm{dj}}=T_5=T_7$ where T_{aj} and T_{dj} are constants, determined by the maximum Acc/Dec ability of the machine tool. The maximum Acc/Dec $a_{\max}$ and $d_{\max}$ can be written as

$$\begin{aligned} a_{\max} &= J_{\max} T_{\mathrm{aj}} \\ d_{\max} &= J_{\max} T_{\mathrm{dj}} \end{aligned} \tag{1}$$

The jerk function $J(t)$ of a whole S-curve Acc/Dec is defined as

$$J(t) = \begin{cases} J_{\max} & t_0\le t\le t_1 \\ 0 & t_0\le t\le t_1 \\ -J_{\max} & t_2\le t\le t_3 \\ 0 & t_3\le t\le t_4 \\ -J_{\max} & t_4\le t\le t_5 \\ 0 & t_5\le t\le t_6 \\ J_{\max} & t_6\le t\le t_7 \end{cases} \tag{2}$$

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From $a(t)=\int_0^t J(t)\mathrm{d}t$, the acceleration function $a(t)$ can be formulated as

$$a(t)=\begin{cases} J_{\max}\tau_1 & t_0\le t\le t_1 \\ a_{\max} & t_1\le t\le t_2 \\ a_{\max}-J_{\max}\tau_3 & t_2\le t\le t_3 \\ 0 & t_3\le t\le t_4 \\ -J_{\max}\tau_5 & t_4\le t\le t_5 \\ -d_{\max} & t_5\le t\le t_6 \\ -d_{\max}+J_{\max}\tau_7 & t_6\le t\le t_7 \end{cases} \quad (3)$$

From $v(t)=\int_0^t a(t)\mathrm{d}t$, the feedrate function $v(t)$ can be formulated as

$$v(t)=\begin{cases} v_s+\frac{1}{2}J_{\max}\tau_1^2, & t_0\le t\le t_1 \\ v_1+a_{\max}\tau_2, & t_1\le t\le t_2,\ v_1=v_s+\frac{1}{2}J_{\max}T_1^2 \\ v_2+a_{\max}\tau_3-\frac{1}{2}J_{\max}\tau_3^2, & t_2\le t\le t_3,\ v_2=v_1+a_{\max}T_2 \\ v_3, & t_3\le t\le t_4,\ v_3=v_{\max}=v_2+\frac{1}{2}J_{\max}T_1^2 \\ v_4-\frac{1}{2}J_{\max}\tau_5^2, & t_4\le t\le t_5,\ v_4=v_3=v_{\max} \\ v_5-d_{\max}\tau_6, & t_5\le t\le t_6,\ v_5=v_{\max}-\frac{1}{2}J_{\max}T_5^2 \\ v_6-d_{\max}\tau_7+\frac{1}{2}J_{\max}\tau_7^2, & t_6\le t\le t_7,\ v_6=v_5-d_{\max}T_6 \end{cases} \quad (4)$$

Similarly, the displacement function $s(t)$ is written as

$$s(t)=\begin{cases} v_s\tau_1+\frac{1}{6}J_{\max}\tau_1^3, & \le t_0\ t\le t_1 \\ s_1+v_1\tau_2+\frac{1}{2}a_{\max}\tau_2^2, & t_1\le t\le t_2,\ s_1=v_sT_j+\frac{1}{6}J_{\max}T_1^3 \\ s_2+v_2\tau_3+\frac{1}{2}a_{\max}\tau_3^2-\frac{1}{6}J_{\max}\tau_3^3, & \\ & t_2\le t\le t_3,\ s_2=s_1+v_1T_2+\frac{1}{2}a_{\max}T_2^2 \\ s_3+v_3\tau_4, & t_3\le t\le t_4,\ s_3=s_2+v_2T_j+\frac{1}{3}J_{\max}T_1^3 \\ s_4+v_4\tau_5-\frac{1}{6}J_{\max}\tau_5^3, & t_4\le t\le t_5,\ s_4=s_3+v_3T_4 \\ s_5+v_5\tau_6-\frac{1}{2}d_{\max}\tau_6^2, & \\ & t_5\le t\le t_6,\ s_5=s_4+v_4T_j-\frac{1}{6}J_{\max}T_5^3 \\ s_6+v_6\tau_7-\frac{1}{2}d_{\max}\tau_7^2+\frac{1}{6}J_{\max}\tau_7^3, & \\ & t_6\le t\le t_7,\ s_6=s_5+v_5T_6-\frac{1}{2}a_{\max}T_6^2 \end{cases} \quad (5)$$

In the real machining, the blocks may not be long enough to ensure the S-curve Acc/Dec including the whole seven sub-phases shown in Fig. 1, but still should have $T_1=T_3$, $T_5=T_7$, $T_1\le T_{aj}$, $T_5\le T_{dj}$, $a_{max}=J_{max}T_1$, and $d_{max}=J_{max}T_5$, so as to avoid sudden change of acceleration. Meanwhile, the ideal S-curve Acc/Dec profile should be time optimal with the limitation of maximum allowable speed, accelerate, and jerk. If the look-ahead function is taken into consideration further, the classification of S-curve Acc/Dec profiles between continuous blocks will be even more complex, it will be discussed in Section 3.3.

2.2 Multiple blocks look-ahead

As shown in Fig. 2, N_1 denotes the first program block; N_2 denotes the second continuous program block and so on. Through the look-ahead function, the CNC system could conduct the trajectory planning of multiple blocks at the same time. The achieved maximum feedrate is improved, and the machining time could be reduced. It is obvious that in a single trajectory calculation, if more program blocks were used, the calculation amount will be increased, but the feedrate profile will be more optimized.

In further considering the S-curve Acc/Dec, the CNC system need to plan the moving speed, acceleration, and jerk along with the whole pre-reading machining tool path of each axes at every interpolation cycle in order to achieve continuous smooth machining with as few corner stop as possible. The following section will introduce a four-level trajectory planning strategy in detail to simultaneously realize the S-curve Acc/Dec and look-ahead function in the multi-axis trajectory plan.

2.3 Four-level trajectory planning strategy

The trajectory planning strategy of multi-axis linkage control could be divided into four levels: position planning, speed planning, acceleration planning, and jerk planning. The general planning principles of the four-level trajectory planning are:

1. The position, velocity, acceleration, and jerk of each moving axis cannot exceed their respective allowable scope, which is determined by the machine tools' physical properties.
2. The synthetic motion speed cannot exceed the command speed, unless there is user-initiated speed adjustment.
3. The planed trajectory should be able to guarantee deceleration stop at any intermediate position without over crossing the end point of the command path.
4. When machining, the actual feedrate of the machine tool should be as far as possible to reach the command federate under the condition of not exceeding the maximum acceleration limit, so as to improve the actual feedrate.
5. For the linear motion of multi-axis, the displacement, speed, acceleration, and jerk of each axis should keep constant proportion relationship at any intermediate position.
6. In each block of once trajectory planning, there is a master planning axis; once trajectory parameters is determined, the following axis' trajectory parameters could get through the linear relation.

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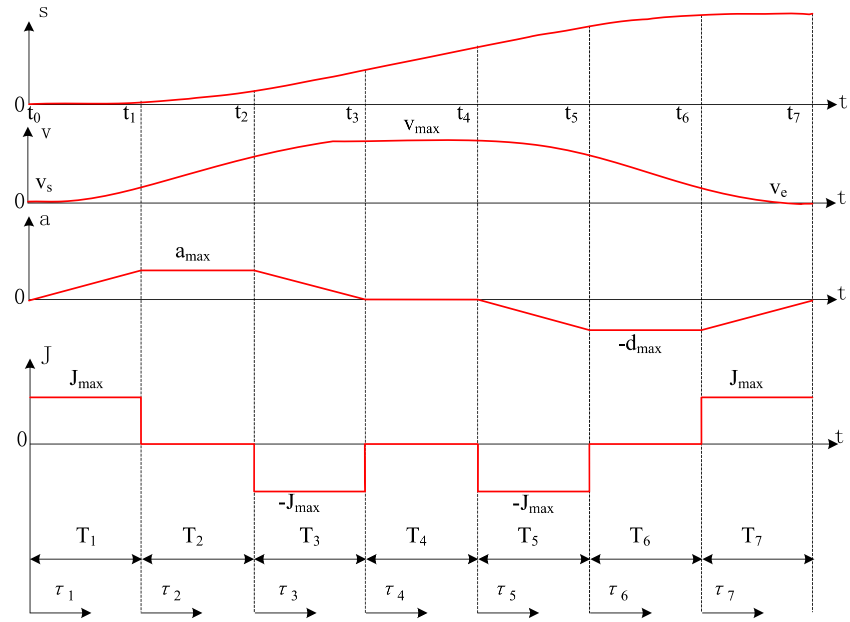

Fig. 1 Typical S-curve Acc/Dec profile

The principles of determining the master planning axis are: The master position planning axis is the axis with maximum displacement; The master speed planning axis is the axis with the maximum ratio of the axis' self moving distance to the axis' maximum allowable speed in the corresponding block; The master acceleration planning axis is the axis with the maximum ratio of the axis's self moving distance to the axis's maximum allowable acceleration in the corresponding block; The master jerk planning axis is the axis with the maximum ratio of the axis's self moving distance to the axis's maximum allowable jerk in the corresponding block.

Figure 3 shows the data flow chart of the whole motion control process with the four-level trajectory planning strategy. The function and general process of each level is described as follows:

1. Position planning: (1) Calculate each axis' displacement of every block; (2) Determine the master position planning axis; (3) Determine the displacement ratio between each axis, which is also the speed, acceleration, and jerk ratio between each axis.
2. Speed planning: (1) Determine the master speed planning axis, assumed to be axis X; (2) Calculate the maximum allowed speed V_{xmax}, uniform motion speed V_{xcmd}, starting speed V_{xs}, constant speed V_{xc}; (3) Calculate the maximum corner speed through the corner speed control algorithm (see Section 3.1) and the maximum allowed end speed V_{xe} through the backward speed planning (see Section 3.2). (4) Determine the type of the master speed planning axis' speed profile through the classification method in forward speed planning and the corresponding time parameters (see Section 3.3).
3. Acceleration planning: (1) Determine the master acceleration planning axis; (2) Determine the maximum acceleration value of the master acceleration axis, which is used to obtain the time parameters of the sub-phases where the jerk value is not zero.
4. Jerk planning: (1) Determine the master jerk planning axis; (2) The jerk value of the master jerk planning axis in the corresponding block's Acc-Acc, Dec-Acc, Acc-Dec, and Dec-Dec process is the axis' maximum allowed jerk value.

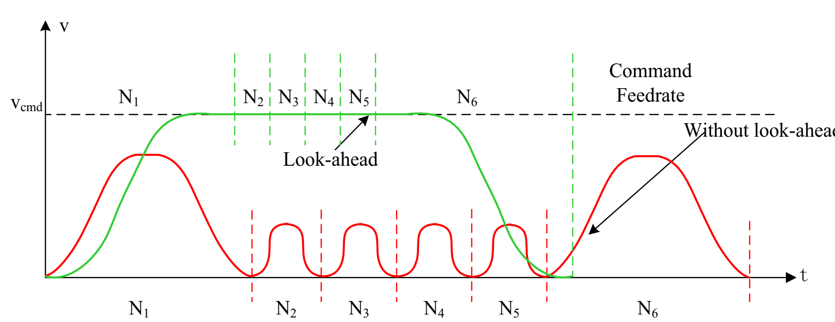

Fig. 2 Advantages of the S-curve Acc/Dec with look-ahead function

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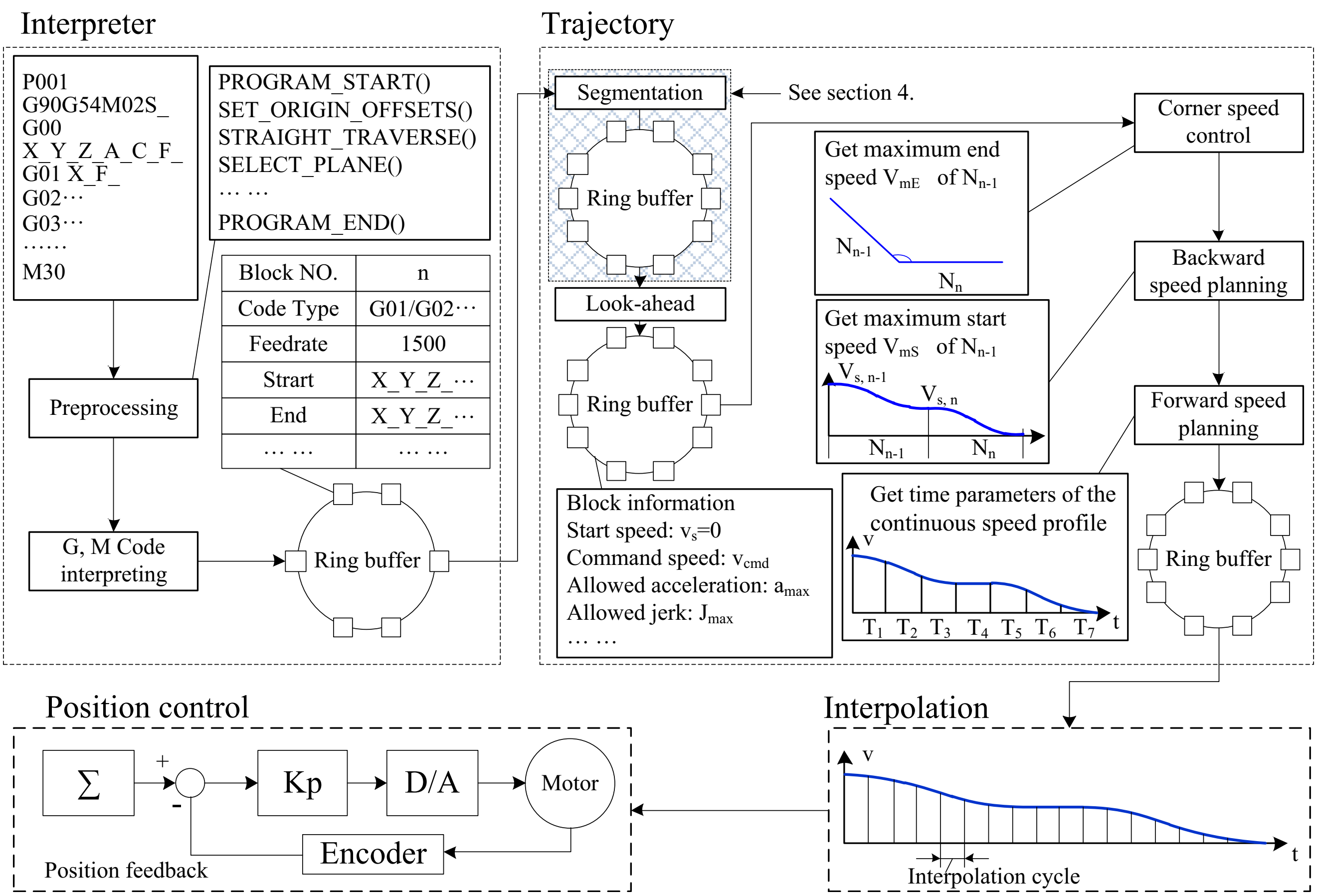
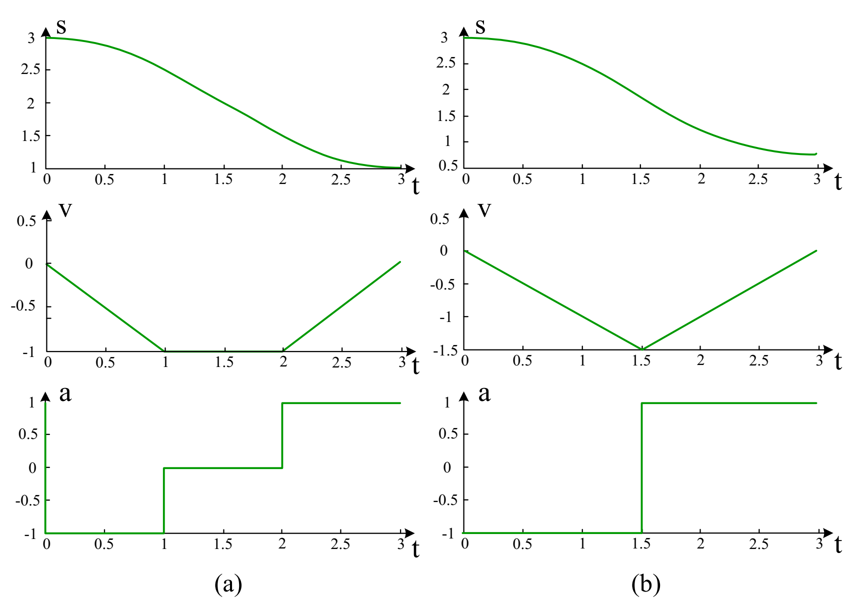


Fig. 3 Data flow chart of the four-level trajectory planning strategy

It is observed that the speed planning is the most important level in the trajectory planning. Through the speed planning, the whole speed profile of the pre-read blocks is obtained and the time parameters of the seven sub-phases are calculated. The look-ahead function and S-curve Acc/Dec is realized through the generation of the continuous S-curve speed profile.

Fig. 4 The speed profile type of S-curve in backward speed planning. **a** Including constant Dec. **b** Not including constant Dec



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3 S-curve speed planning with look-ahead function

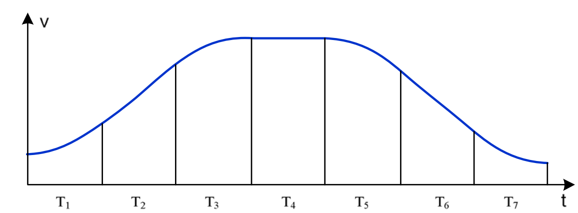
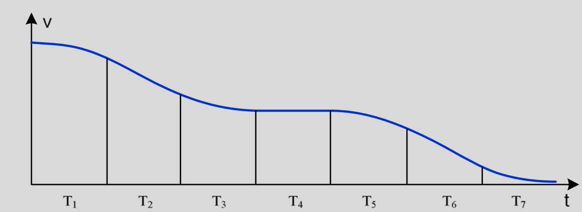
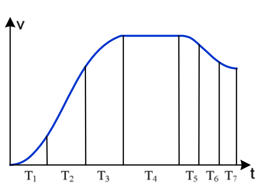
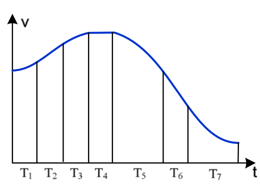
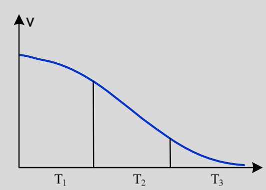
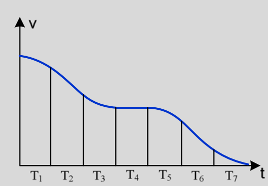
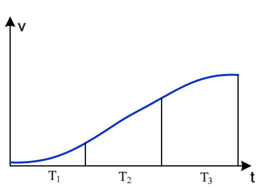
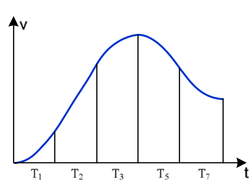
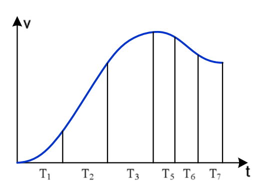
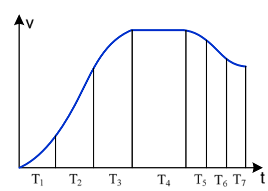
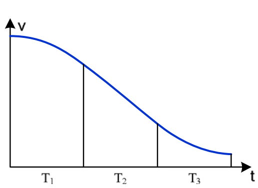
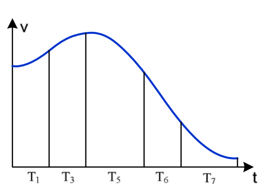
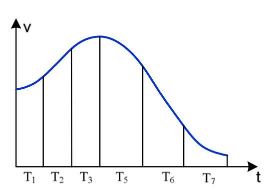
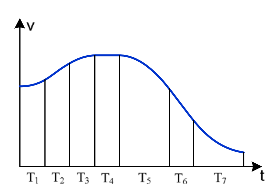
3.1 Corner speed control method

Due to the direction and magnitude of two successive blocks are most likely to change, the corner speed needs to be properly controlled to avoid the speed sudden change, which will cause machine vibration. In general, the axes of the machine tool are orthogonal to each other, the control of the joint acceleration of the machine tool can be simplified into the control of the master acceleration planning axis, as the master acceleration planning axis is the axis that will most likely exceed the allowed acceleration.

$$\begin{aligned}
v_{X1} &= F_1\frac{X_{E1}-X_{S1}}{L_1}, v_{Y1} = F_1\frac{Y_{E1}-Y_{S1}}{L_1}, v_{Z1} = F_1\frac{Z_{E1}-Z_{S1}}{L_1},\\
v_{A1} &= F_1\frac{A_{E1}-A_{S1}}{L_1}, v_{B1} = F_1\frac{B_{E1}-B_{S1}}{L_1}, v_{C1} = F_1\frac{C_{E1}-C_{S1}}{L_1},\\
v_{X2} &= F_2\frac{X_{E2}-X_{S2}}{L_2}, v_{Y2} = F_2\frac{Y_{E2}-Y_{S2}}{L_2}, v_{Z2} = F_2\frac{Z_{E2}-Z_{S2}}{L_2},\\
v_{A2} &= F_2\frac{A_{E2}-A_{S2}}{L_2}, v_{B2} = F_2\frac{B_{E2}-B_{S2}}{L_2}, v_{C2} = F_2\frac{C_{E2}-C_{S2}}{L_2},
\end{aligned} \tag{6}$$

For convenience of explanation, take six axes' linear continuous motion as an example; the first block is N_1 and the

Table 1 Classification of speed profile in forward speed planning

Main Type I		Main type II	
			
$v_s \le v_t$		$v_s > v_t$	
Type I-1	Type I-2	Type II-1-a	Type II-1-b
			
$v_s \le v_{et}$	$v_s > v_{et}$	$T_1=T_3$, $T_4=T_5=T_6=T_7=0$	$T_1=T_3$, $T_5=T_7$, $T_4\neq0$
Type I-1-a	Type I-1-b	Type I-1-c	Type I-1-d
			
$T_1=T_3, T_4=T_5=T_6=T_7=0$	$T_1=T_3$, $T_5=T_7$, $T_4=0$, $T_6=0$	$T_1=T_3$, $T_5=T_7$, $T_4=0$, $T_6\neq0$	$T_1=T_3$, $T_5=T_7$, $T_4\neq0$
Type I-2-a	Type I-2-b	Type I-2-c	Type I-2-d
			
$T_1=T_2=T_3=T_4=0, T_5=T_7$	$T_1=T_3$, $T_5=T_7$, $T_4=0$, $T_2=0$	$T_1=T_3$, $T_5=T_7$, $T_4=0$, $T_2\neq0$	$T_1=T_3$, $T_5=T_7$, $T_4\neq0$

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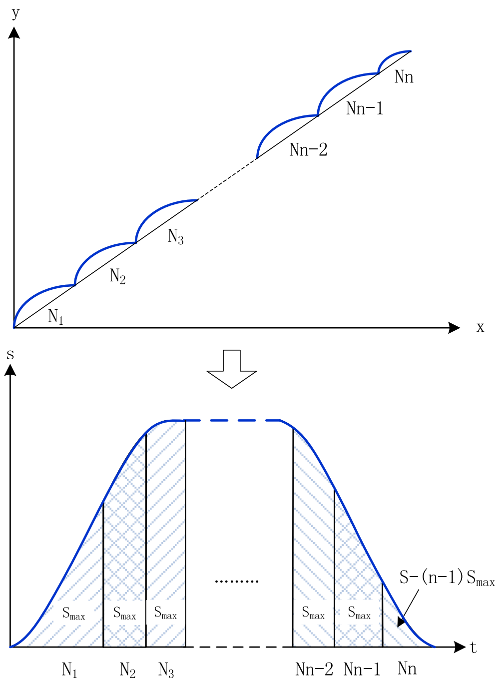

Fig. 5 Nanoscale trajectory planning method using long blocks segmentation

next block is N_2. The start point and the end point of N_1 are $(X_{S1},Y_{S1},Z_{S1},A_{S1},B_{S1},C_{S1})$ and $(X_{E1},Y_{E1},Z_{E1},A_{E1},B_{E1},C_{E1})$, respectively. The start point and the end point of N_2 are $(X_{S2},Y_{S2},Z_{S2},A_{S2},B_{S2},C_{S2})$ and $(X_{E2},Y_{E2},Z_{E2},A_{E2},B_{E2},C_{E2})$, respectively. Besides, the feedrate of N_1 is F_1, and the feedrate of N_2 is F_2. In this case, the speed of blocks N_1 and N_2 in the direction of each axis are given by Eq. (6). Where $v_{i,j}(i=X,Y,Z,A,B,C;j=1,2)$ is the speed of each axis. $L_i(i=1,2)$ is the length of block N_i.

The speed variation $\Delta v_i(i=X,Y,Z,A,B,C)$ of each axis between two continuous blocks is as follows:

$$\Delta v_X=(v_{X2}-v_{X1}),\Delta v_Y=(v_{Y2}-v_{Y1}),\Delta v_Z=(v_{Y2}-v_{Y1}),\\ \Delta v_A=(v_{A2}-v_{A1}),\Delta v_B=(v_{B2}-v_{Y1}),\Delta v_B=(v_{C2}-v_{C1}), \tag{7}$$

The maximum allowable speed variation of each axis is defined as $\Delta v_{mi}(i=X,Y,Z,A,B,C;j=1,2)$. In Eq. (9), Q is the maximum speed variation degree of all the axes.

$$Q=\max\left\{\frac{\Delta v_X}{\Delta v_{mX}},\frac{\Delta v_Y}{\Delta v_{mY}},\frac{\Delta v_Z}{\Delta v_{mZ}},\frac{\Delta v_A}{\Delta v_{mA}},\frac{\Delta v_B}{\Delta v_{mB}},\frac{\Delta v_C}{\Delta v_{mC}}\right\} \tag{8}$$

Obviously, when $Q\leq 1$, the speed of each axis will not exceed the maximum allowable speed of each axis at the corner; When $Q>1$, the speed of one or more axes exceed the responding axis' maximum allowable speed, which means the speed of each axis needs to be limited. The limited speed v_{imE1} of each axis at the end of block N_1, and the maximum allowed speed v_{imS2} of each axis at the beginning of block N_2 can be computed as

$$\begin{cases} v_{ime1}=\dfrac{v_{i1}}{Q} \\ v_{ims2}=\dfrac{v_{i2}}{Q} \end{cases} \quad i=X,Y,Z,A,B,C \tag{9}$$

From Eq. (9), the speed variation of each axis at the corner could be efficiently controlled so as to realize high-speed machining of continuous micro line blocks.

3.2 Backward speed planning

In the look-ahead mode, the machine tool should be able to smoothly stop at any position along the machining path without running deviation, so the maximum end speed $v_{imej}(i=X,Y,Z,A,B,C;j=1,2,3,\ldots n)$ of each pre-read block needs to be calculated firstly with the initial condition $v_{imen}(i=X,Y,Z,A,B,C;j=1,2,3,\ldots n)=0$, i.e., the end speed of the last block N_n is 0. It is a reverse calculation process, starts from the last block N_n, and ends at the first block N_1. Assume the master speed planning axis' maximum end speed of block N_j is v_e, and the moving distance is s, v_s is the maximum start speed which needs to be calculated through S-curve profile. As shown in Fig. 4a,

Table 2 Simulation data of four-level trajectory planning strategy

N_i	s_i (m)	v_{imax} (m/s)	v_{ie} (m/s)	J_{imax} (m/s^3)	Standard axis vector	Type
N_1	0.053	0.178	0.178	2.236	(0.447, 0.447, 0.447, 0.447, 0.447)	I-1-a
N_2	0.593	0.581	0.476	5.136	(0.195, 0.487, 0.234, 0.779, 0.253)	I-1-b
N_3	0.455	0.689	0.633	2.713	(0.368, 0.553, 0.737, 0.123, 0.000)	I-1-c
N_4	0.280	0.691	0.659	1.597	(0.626, 0.188, 0.438, 0.250, 0.564)	I-1-d
N_5	0.382	0.659	0.536	6.307	(0.159, 0.190, 0.349, 0.713, 0.555)	I-2-a
N_6	0.252	0.573	0.474	1.641	(0.609, 0.183, 0.548, 0.427, 0.335)	I-2-b
N_7	0.402	0.539	0.339	3.309	(0.302, 0.332, 0.635, 0.363, 0.514)	I-1-c
N_8	0.342	0.404	0.203	3.348	(0.299, 0.777, 0.478, 0.239, 0.149)	I-1-d
N_9	0.078	0.203	0.123	4.082	(0.245, 0.612, 0.392, 0.441, 0.465)	II-1-a
N_{10}	0.081	0.123	0.000	3.341	(0.299, 0.299, 0.718, 0.359, 0.419)	II-1-b

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an S-curve acceleration process generally contains Acc-Dec, constant Dec, Dec-Dec, but when the moving distance is not long enough, as shown in Fig. 4b, the constant Dec sub-phase will not appear. Assume s_1 is the critical value under which the S-curve profile only contains the Acc-Acc and Dec-Acc sub-phase. The value of s_1 could be calculated through Eqs. (4) and (5) with the condition $T_5=T_7=T_{\rm dj}$. When $s>s_1$, the S-curve profile contains constant Dec sub-phase, the moving distance s of the master speed planning axis can be written as:

$$s=\frac{1}{2}J_{\max}T_{\rm dj}T_6^2+\left(v_{\rm e}+\frac{3}{2}J_{\max}T_{\rm dj}^2\right)T_6+2v_{\rm e}T_{\rm dj}+J_{\max}T_{\rm dj}^3 \quad (10)$$

Set

$$\begin{cases} A=\frac{1}{2}J_{\max}T_{dj} \\ B=v_e+\frac{3}{2}J_{\max}T_{dj}^2 \\ C=2v_eT_{dj}+J_{\max}T_{dj}^3-s \end{cases} \quad (11)$$

Where A, B, and C can be obtained directly, then T_6 can be formulated as

$$T_6=\frac{-B+\sqrt{B^2-4AC}}{2A} \quad (12)$$

v_s can be computed by Eq. (4). When $s\le s_1$, there are $T_6=0$, $T_5=T_7$, and

$$s=J_{\max}T_5^3+2v_{\rm e}T_5 \quad (13)$$

Eq. (13) is a cubic equation, according to the method for solving cubic equation, set

$$\Delta=27J_{\max}^2s+\sqrt{4(6J_{\max}v_{\rm e})^3+\left(27J_{\max}^2s\right)^2} \quad (14)$$

We can get

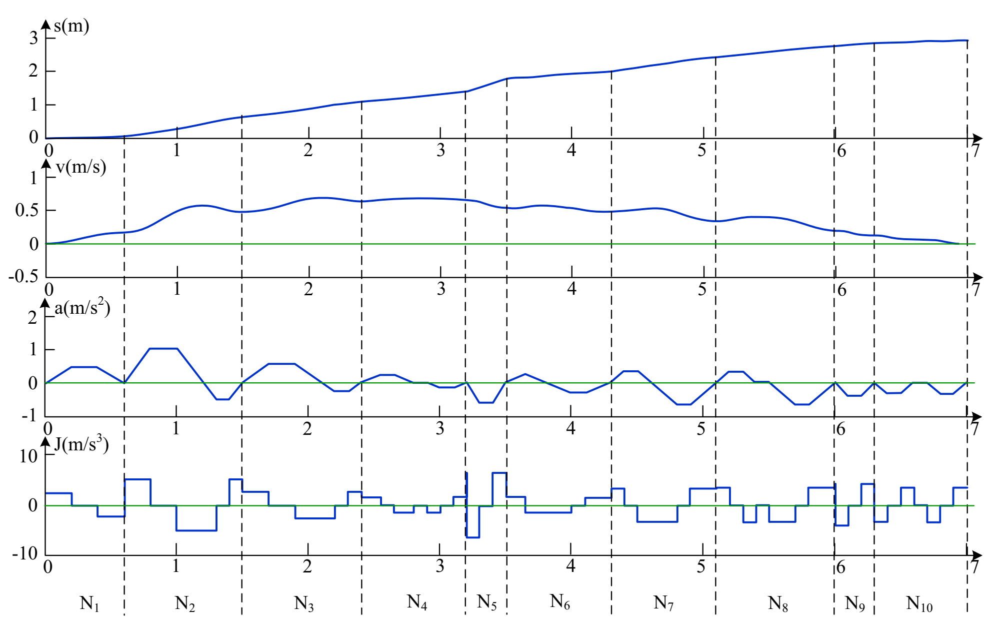
$$T_5=-2v_{\rm e}\left(\frac{2}{\Delta}\right)^{\frac{1}{3}}+\frac{1}{3J_{\max}}\left(\frac{\Delta}{2}\right)^{\frac{1}{3}} \quad (15)$$

Then the maximum start v_s of the current block can be calculated by Eq. (4), through the backward speed planning method elaborated in Section 2.3, the maximum end speed v_r of the last block should be less than v_s.

3.3 Forward speed planning

In forward speed planning, the task is to compute the time parameters $T_k(k=1\cdots7)$ of each block from N_1 to block N_n, with the known conditions, i.e., s, v_s, v_t, and $v_{\rm et}(v_t\ge v_{\rm et})$. As the displacement, start speed, maximum end speed of each axis is various; the speed profiles will not be a complete 7-segment process in general. As shown in Table 1, we give a classification method of the speed profile in forward speed planning through the master planning axis' displacement gradually increasing way. Firstly, the speed profile is classified into two main types by the magnitude of the start speed v_s and target speed v_t: when $v_s\le v_t$, called Type I, and when $v_s\ge v_t$, called Type II; Secondly, Type I can be divided into two subtypes: Type I-1 when $v_s\le v_{\rm et}$ and Type I-2 when $v_s>v_{\rm et}$; Finally, Type I-1, Type I-2, and Type II can be further classified into even

Fig. 6 Simulation result of four-level trajectory planning strategy



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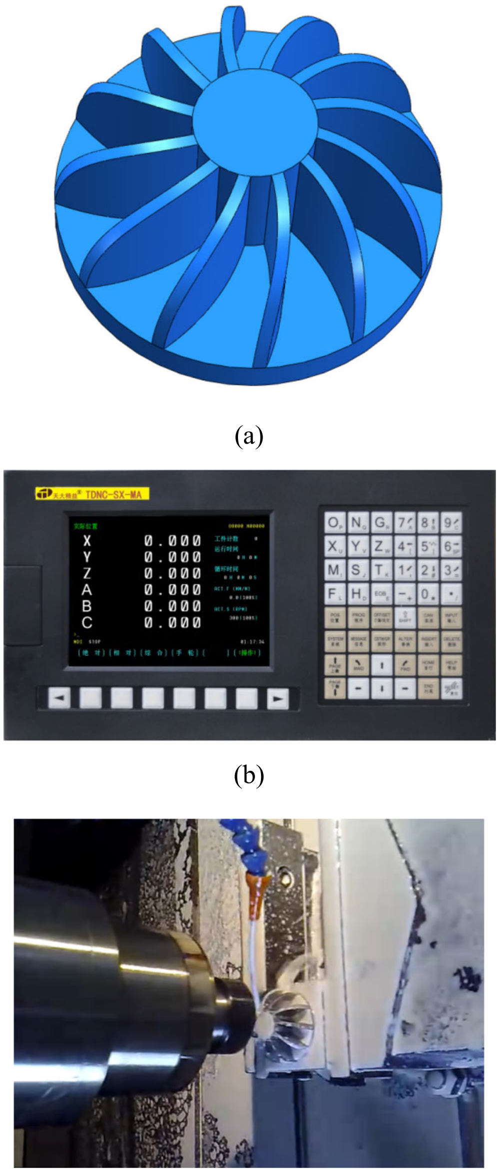

(a)

(b)

(c)

Fig. 7 Impeller machining experiment applying nanoscale trajectory strategy. **a** Impeller CAD model. **b** The CNC system used for the machining. **c** The machined part

more subtypes I-1-a to I-1-d, I-2-a to I-2-d, and II-1-a to II-1-b by the way of gradually increasing the master planning axis' displacement. All of the speed profiles of the involved types are shown in Table 1. According to the speed profiles and Eqs. (2) to (5), the time parameters $T_k(k=1\cdots7)$ can be eventually computed by solving the responding equation set.

4 Nanoscale trajectory method

In order to solve the computing precision control problem caused by the large variation range of the length of the continuous blocks and make the whole trajectory planning strategy have better cross-platform features, we analyzed the four-level planning processes in detail, and found that the length of the blocks is the key factor influencing the computing precision. When the blocks are short enough such as only a few microns, the calculation result can be expressed with high precision, can even reach the nanometer level. So, the nanoscale trajectory planning method can be realized by using long blocks segmentation.

With detailed calculation, the length unit of the blocks was firstly set to 1 in 2^7 nm, so as to ensure minimal accuracy loss. For 32-bit length data, the length range would be -1×2^{24}~1×2^{24} nm, i.e., −16.777~16.777 mm. As shown in Fig. 5, in order to make the length of all the blocks fall into the range, the long blocks were divided into short blocks. As the segmentation could be executed before the look-ahead function, the trajectory planning method with look-ahead and S-curve Acc/Dec did not need other changes besides processing more blocks.

The proposed nanoscale trajectory planning method can bring the following advantages:

1. With ordinary 32-bit processor, the length data of CNC blocks can be represented with high accuracy, which can reach less than 0.01 nm.
2. Improve the input data accuracy of the interpolation module; Make it possible to realize nanoscale interpolation on ordinary 32-bit industrial processors.
3. Although the number of blocks increases, but in fact, when the CNC system do the trajectory planning, position control of long blocks will appear more calculated idle time. Therefore, the segmentation of long blocks will not really result in an increase in the processor load, but can improve

Table 3 Statistics of the four comparing groups

	Using nanoscale segmentation			Not using nanoscale segmentation		
	Machining time	Average CPU utilization (%)	Maximum CPU utilization (%)	Machining time	Average CPU utilization (%)	Maximum CPU utilization (%)
Using look-ahead	58 min 30 s	46	68	58 min 27 s	41	66
Not using look-ahead	62 min 20 s	33	45	62 min 15 s	28	44

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computation efficiency and achieve rational allocation of computing resources.

4. The hardware performance requirements and design difficulty will decrease, thus reducing design and manufacturing cost of high-precision motion control systems.

5 Simulation and experimental results

The proposed four-level trajectory planning strategy was firstly validated through continuous S-curve Acc/Dec simulation of a series of typical continuous micro line segments which contains all the speed profile types mentioned in Section 3.3. The simulation data is listed in Table 2, in which s_i is the length of line segment N_i, v_{imax} is the corresponding maximum speed reached in the line segment N_i, v_{ie} is the end speed at the end of line segment N_i, J_{imax} is the max jerk reached in the line segment N_i. s_i, v_{imax}, v_{ie}, and J_{imax} are the resultant vectors by five-axis vectors; the stand axis vectors and the type of N_i are all listed in the table.

As shown in Fig. 6, the displacement, speed, acceleration, and jerk profiles of the continuous micro line segments are drawn together. From the speed profile, it can be seen that the transition between each two continuous blocks is smooth. There is no sudden acceleration change in the acceleration profile. The jerk of the whole moving path is successfully controlled under the maximum allowable jerk, which could effectively reduce the machining shocks and improve the surface quality.

To further verify the effectiveness of the proposed nanoscale trajectory planning method, an experiment including four comparing groups were made on a CNC milling machine. An impeller CAD model shown in Fig. 7a was used to generate the NC programs. The whole nanoscale trajectory planning strategy with the S-curve Acc/Dec and look-ahead function was integrated into an embedded CNC system based on ARM and DSP, which is shown in Fig. 7b. The machining process and the part are shown in Fig. 7c.

The first group used both the nanoscale segmentation method and look-ahead function; The second group just uses the nanoscale segmentation method; The third group just used the look-ahead function; The fourth group did not use the nanoscale segmentation method nor look-ahead function; In the machining process, the machining time, average CPU utilization, and maximum CPU utilization of each group were recorded; the statistics are shown in Table 3.

From the statistics, we can find that when using the look-ahead function, the CPU utilization increased about 10 %, while the machining time was significantly reduced, from 62 min 20 s to 58 min 30 s; When using the nanoscale segmentation method, the average CPU utilization increased about 5 %; however, the maximum CPU utilization just increased by 1–2 %. So, the experiment result shows that the look-ahead function can effectively reduce the machining time, and the nanoscale segmentation method will result in comparatively obvious average CPU utilization increase, but little maximum CPU utilization increase, which can effectively increase the internal calculation accuracy and make more reasonable allocation of CPU computing resources.

6 Conclusion

Multiple blocks look-ahead, corner speed control, and S-curve Acc/Dec are the key issues of the high-speed and high-precision machining technology; this paper presents a four-level trajectory planning strategy which can realize continuous S-curve Acc/Dec control with jerk limitation. Firstly, a common corner speed control method used for multiple axes was proposed to compute the maximum allowed speed between two continuous blocks. Secondly, the profiles in the forward speed planning of the S-curve Acc/Dec control were classified into 10 types, which can cover every possible Acc/Dec situation. Thirdly, for the accuracy loss caused by the large variation range of the length of the continuous blocks, a long blocks segmentation method was designed to realize nanoscale trajectory planning on an ordinary 32-bit processor. Finally, the efficiency and practicality of the overall nanoscale trajectory planning strategy was first successfully validated by continuous S-curve Acc/Dec simulation of a series of typical continuous micro line segments, and then by machining a group of identical impellers.

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